upper Parts, under which the Colours have appear'd. I
have taken notice of this so very often, that I can hard-
ly look upon it to be accidental, and if it should prove
true in general, it will bring the disquisition into a nar-
row compass; for it will shew that this Effect depends
upon some Property, which the Drops retain, whilst
they are in the upper part of the Air, but lose as they
come lower, and are more mix'd with one another.

Petworth, *Oct.* 13.
  1722.

V. *A Letter to Dr.*Jurin, *Coll. Med.* Lond. *Soc.*
& Secr. R. S. *concerning the abovementioned Ap-*
pearance in the Rainbow, *with some other* Refle-
ctions on the same Subject. By Henry Pem-
berton, *M. D.* R. S. S.

S I R,

UPON your communicating to me the curious
Observations, your Friend Dr. *Langwith* had
made on the Rainbow, I inform'd you those Appear-
ances might, I thought, be explain'd by the Disco-
veries, the Great Sir *Isaac Newton* had made in
the Subject of Light and Colours, in his wonderful
Treatise of *Optics*. As you seemed not displeased
with what I mentioned to you in relation to this Mat-
ter by word of mouth, you desired that I would set
down in writing my Thoughts thereupon, which I
have here accordingly done in the following manner.
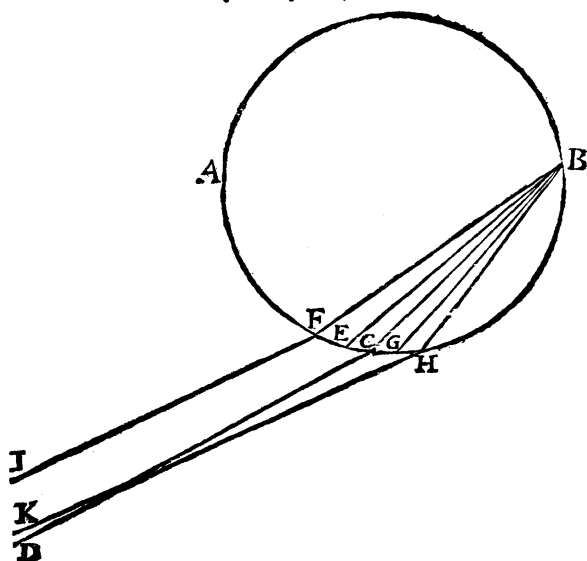
Let A B reprefent a Drop of Rain, B the Point from
whence the Rays of any determinate Species being
reflected to C, and afterwards emerging in the Line
C D, do proceed to the Eye, and caufe the Appearance
of that Colour in the Rainbow, which appertains to
this Species.   It is obferved by Sir *Ifaac Newton* [a],
that in the Reflection of Light, befides what is re-
flected regularly, fome fmall part of it is irregularly
fcattered every way.   So that from the Point B, be-
fides the Rays that are regularly reflected from B to C,
fome fcattered Rays will return in other Lines, as in
B E, B F, B G, B H, on each Side the Line B C.   Fur-
ther it muft be noted from Sir *Ifaac Newton* [b], that
the Rays of Light in their Paffage from one Superficies
of a refracting *Medium* to the other undergo alternate

---

[a] Optics, *Book* II. *Part* 4,   [b] Ibid. *Part* III. *Prop.* xij.

Fits

Fits of eafy Tranfmiffion and Reflection, fucceeding each other at equal Intervals ; infomuch that if they reach the further Superficies in one fort of thofe Fits, they fhall be tranfmitted ; if in the other kind of them, they fhall rather be reflected back. Whence the Rays that proceed from B to C, and emerge in the Line C D, being in a Fit of eafy Tranfmiffion, the fcattered Rays that fall at a fmall Diftance without thefe on either fide, (fuppofe the Rays, that pafs in the Lines B E, B G) fhall fall on the Surface in a Fit of eafy Reflection, and fhall not emerge ; but the fcattered Rays,that pafs at fome Diftance without thefe laft, fhall arrive at the Surface of the Drop in a fit of eafy Tranfmiffion, and break through that Surface   Suppofe thefe Rays to pafs in the Lines B F, B H ; the former of which Rays fhall have had one Fit more of eafy Tranfmiffion, and the latter one Fit lefs, than the Rays that pafs from B to C. Now both thefe Rays, when they go out of the Drop, will proceed by the Refraction of the Water in the Lines F I, H K, that will be inclined almoft equally to the Rays incident on the Drop, that come from the Sun, but the Angles of their Inclination will be lefs than the Angle, in which the Rays emerging in the Line C D are inclined to thofe incident Rays. And after the fame manner Rays fcattered from the Point B, at a certain Diftance without thefe, will emerge out of the Drop, while the intermediate Rays are intercepted; and thefe emergent Rays will be inclined to the Rays incident on the Drop in Angles ftill lefs than the Angles, in which the Rays F I and H K are inclined to them ; and without thefe Rays will emerge other Rays, that fhall be inclined to the incident Rays in Angles yet lefs. Now by this means will be formed of every kind of Rays, befides the principal Arch which goes to the Forma-

tion

tion of the Rainbow, other Arches, within every one of the principal, of the fame Colour, though much more faint : and this for divers Succeffions, as long as thefe weak Lights, which in every Arch grow more and more obfcure, fhall continue vifible. Now as the Arches produced by each Colour will be varioufly mixed together, the diverfity of Colours obferved by Dr. *Langwith* may well arife from them.

The precife Diftances between the principal Arch of each refpective Colour and thefe fainter correfpondent Arches depend on the Magnitude of the Drops of Rain. In particular, the fmalleft Drops will make the fecondary Arches of each Species at the greateft Diftance from their refpective principal, and from each other. Whence, as the Drops of Rain increafe in falling, thefe Arches near the Horizon by their great Nearnefs to their refpective principal Arches become invifible.

A ND now, Sir, we are upon the Rainbow, I fhall here take the Freedom of fetting down two Propofitions, which I have formerly confidered, relating to this Subject. For the greater Brevity I fhall deliver them under the Form of Porifms ; as, in my Opinion, the Ancients called all Propofitions treated by Analyfis only.

PROPOSITION I.

In a given refracting Circle, whofe refracting Power is given, the Ray is given in Pofition, which paffing parallel to a given Diameter of the Circle is refracted by that Circle to a Point given in the Circumference of it.

Let

Let A B C D be the given Circle, the given Diameter
A C, and given Point G ; and let the Ray E F, parallel
to A C, be refracted to G.   I fay E F is given in Po-
fition.

Produce E F to H, and draw the Diameter F I, draw-
ing likewife I K H, I G.   Then is H F I the Angle of
Incidence, and G F I the refracted Angle ; fo that I H
being perpendicular to F H and I G perpendicular
to F G, I H is to I G as the Sine of the Angle of
Incidence to the Sine of the refracted Angle, and
the *Ratio* of I H to I G is given, as likewife the
Ratio of I K to I G.   Therefore I K being perpen-
dicular to A C the Point I is in a Conic Section given
in Pofition, whofe Axis is perpendicular to A C,
and one of its *Foci* is the Point G [a].   Confequently
the Points I and F are given, and laftly the Ray E F
given in Pofition.

_____

[a] *See* Papp. l. 7. prop. 238.   Milnes Conic. part. 4. prop. 9.

DETER-

DETERMINATION.

IT is evident, that this conic Section, may either cut the Circle in two Points, touch it in one Point, or fall wholly without it. Therefore let the Section touch the Circle in the Point I, and let I L touch both the Section and the Circle in the same point I. Then G L being joined, the Angle under I G L on ac-

count of the conic Section is a right one [a], so that F G L is one continued right Line, and I F is to I L as F G to G I; as likewise, M being the Center of the Circle, M I to I L, or F H to H I, as F G to twice G I, because M I is to I F as G I to twice G I. Hence by Permutation F H is to F G as H I to twice G I; that is, as the Sine of the Angle of Incidence to twice the Sine of the refracted Angle.
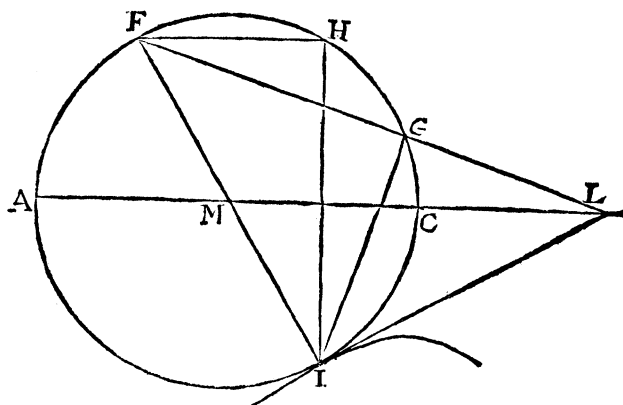
---

[a] De la Hire Conic. lib. 8. prop. 23.

Moreover

(251)

Moreover F H being to H I as F G to twice G I, the Square of F H will be to the Square of H I, as the Square of F G to four times the Square of G I. Therefore, by Compofition, as the Square of F H to the Square of F I or of A C, fo is the Square of F G to the Square of F I together with three times the Square of G I, and fo likewife is the Excefs of the Square of F G above the Square of F H, which equals the Excefs of the Square of I H above the Square of I G, to three times the Square of G I; for as one Antecedent to one Confequent, fo is the difference of the Antecedents to the difference of the Confequents. Hence in the laft place, the Square of half F H will be to the Square of A M, as the Excefs of the Square of I H above the Square of I G to three times the Square of I G, or as the Excefs of the Square of the Sine of Incidence above the Square of the Sine of Refraction, to three times the Square of the Sine of Refraction.

Another

Another DETERMINATION.

Draw the Diameter G O and the Tangent O P, meeting G F produced in Q: then the Angle under I F G is equal to the Angle under O G F, the Angle
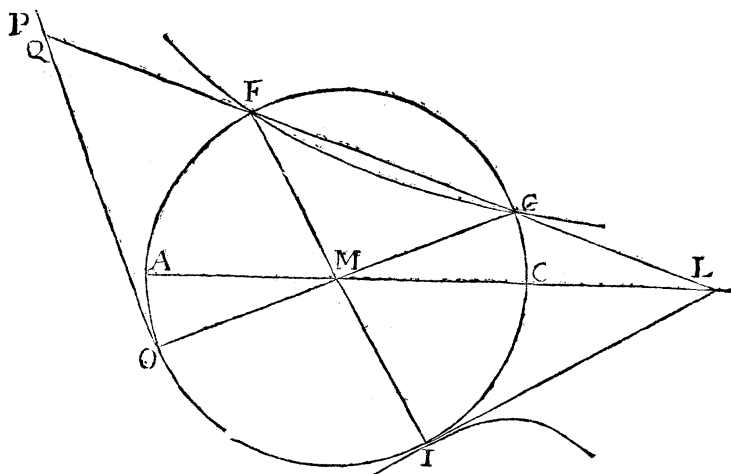
under F I L equal to that under G O Q, both being right, and F I is equal to G O ; whence the Triangles G O Q, F I L are fimilar and equal ; fo that G Q is equal to F L, and the Point F in an *Hyperbola* paffing through G, whofe Afymptotes are A C and O P [a].

---

[a] Apoll. Conic. l. 2. prop. 8.

PROPO-

PROPOSITION II.

A refracting Circle and its refracting Power be-
ing given, the Ray is given in Position, which, paf-
fing parallel to a given Diameter of the Circle, after
its Refraction, is fo reflected from the farther Sur-
face of the Circle, as to be inclined to its incident
Courfe in a given Angle.

Let A B C D be the given Circle ; let A C be the
given Diameter, E F the incident Ray parallel to it,
which being refracted into the Line F G fhall fo be re-
flected from the Point G in the Line G H, that E F
and H G being produced, till they meet in I, the
Angle under E I H fhall be given.

Let K be the Center of the Circle, and K F, K G be
joined ; let the Semidiameter L K be parallel to the
S   refracted

refracted Ray F G, and M K being taken to the Semi-diameter of the Circle in the *Ratio* of the Sine of Incidence to the Sine of Refraction; let L M be joined, and lastly make the Angle under K M N equal to half the given Angle under E I H. This being done, if F G be produced to O, F O shall be to K O as the Sine of the Angle of Incidence to the Sine of the refracted Angle, that is as M K to K L; in so much that K L being parallel to F O, and the Angle under M K L equal to that under F O K, the Angle under M L K shall be equal to that under F K O, and the Angle under K M L equal to that under K F O equal to that under F G K or half that under F G H, whence the Angle under K M N being equal to half the Angle under F I H, the residuary Angle under N M L will be equal to half the Angle under I F G or to half that under M K L. Therefore L C being drawn, the Angle under L M N will be equal to that under M C L; and in the last place, if M C be divided into two equal Parts in P, and P Q R be drawn parallel to C L, the Angle under Q M R will be equal to that under R P M, and the Triangles Q M R, M P R similar, so that the Rectangle under P R Q shall be equal to the Square of M R. Whence R L being equal to M R, the Point L shall be in an equilateral *Hyperbola*, touching the Line M N in the Point M, and having the Point P for its Center [a]. But this *Hyperbola* is given in Position, and consequently the Point L, the Angle under M L K, and the equal Angle under C K F will be given, and therefore the Ray E F is given in Position.

---

[a] Apoll. Conic. lib. 1. prop. 37. *compared with* lib. 7. prop. 23.

DETERMINATION.

Let the *Hyperbola* touch the Circle in the Point L, and let their common Tangent be L S; draw L T parallel to M N, fo as to be ordinately applied in the *Hyperbola* to the Diameter C M. Whence L S touching the *Hyperbola* in L, P T will be to T L as T L to T S [a], and the Angle under T S L equal to that under T L P, but as the Angle under S C L is equal to that under N M L, the fame is equal to the Angle under T L M ; therefore the Angle under S L C is equal to the Angle under M L P. Farther, M L being produced to V and V C joined, the Angle under L V C is

equal to that under S L C, by reafon that L S touches the Circle in L ; hence the Angles under L V C and under M L P are equal, L P, V C are parallel, and

---

[a] Apoll. Conic. lib. 1. prop. 37. *compared with* lib. 7. prop. 23.

M P being equal to P C, M L is equal to L V ; and K W being let fall perpendicular to LV, M W is equal to three times L W. But now if the incident Ray E F be produced to X, the Angle under M L K being equal to that under C K F, or to that under E F K, F X ſhall be equal to LV, equal to twice L W ; and the Angle under K M L being equal to that under K F G ; ſince K W is perpendicular to M W, F G ſhall be to twice M W as M K to K F, or as the Sine of Incidence to the Sine of Refraction : whence M W being equal to three times L W, F X ſhall be to F G as the Sine of Incidence to three times the Sine of Refraction.

Moreover, M W being equal to three times L W, the Square of M W will be equal to nine times the Square of L W, and the Rectangle under V M L, or the Rectangle under C M A, that is, the Exceſs of the Square of K M above the Square of K A, will be equal to eight times the Square of L W ; therefore the Square of L W or the Square of half F X will be to the Square of K L, or of K A, as the Exceſs of the Square of K M above the Square of K A to eight times the Square of K A, that is, as the Exceſs of the Square of the Sine of Incidence, above the Sine of Refraction to eight times the Square of the Sine of Refraction.

Another

Another *DETERMINATION.*

Draw A Y parallel to M N, and A Z parallel to
M V : then is the Angle under Y A Z, equal to that
under L M N, which is equal to that under L C A ;
whence the Arches A L, Y Z are equal ; but the

Arches A L, V Z are likewise equal, because L V, A Z
are parallel, therefore Y V being joined, and L Γ drawn
perpendicular to A C, the Chord V Y ſhall be the
double of L Γ ; but V Δ being likewiſe let fall per-
pendicular to A C, becauſe M V is the double of M L,
V Δ ſhall be the double of L Γ ; and therefore V Δ
and V Y ſhall be equal ; whence the point V ſhall be
in a *Parabola*, whoſe *Focus* is the Point Y, its Axis
perpendicular to A C, and the *Latus rectum*, belong-
ing to that Axis, equal to twice the perpendicular let
fall from Y upon A C [a]. But if K V be joined, the

_____

[a] Vide de la Hire Sect. Conic. lib. 8. prop. 1, 3.

Angle under L K V is equal to twice the Complement
to a right Angle of the Angle under K L V, which is
equal to the Angle of Incidence, and exceeds the re-
fracted Angle by the Angle under A K L.

T H E Determinations of thefe two Propofitions,
have relation to the firft and fecond Rainbow ; thofe
of the firft Propofition refpecting the interior, and thofe
of the fecond the exterior. The firft Determinations
of thefe two Propofitions affign the Angles, under
which each Rainbow will appear in any given refract-
ing Power of the tranfparent Subftance, by which they
are produced ; the latter Determinations of thefe Pro-
pofitions teach how to find the refracting Power of the
Subftance, from the Angles under which the Rain-
bows appear ; the Angle under C M G, in the Deter-
minations of the firft Propofition, being half the Angle
which meafures the Diftance of the interior Bow from
the Point oppofite to the Sun ; and in the Determi-
nations of the fecond Propofition, the Angle under
C M N is half the Complement to a right Angle of
half the Angle that meafures the Diftance of the exterior
Bow, from the Point oppofite to the Sun. But where-
as thefe latter Determinations require folid Geometry,
it may not be amifs here to fhew how they may be
reduced to Calculation, feeing the Obfervation of thefe
Angles, as the learned Dr. *Halley* has already re-
mark'd [a], affords no inconvenient Method of find-
ing the refracting Power of any Fluid, or indeed of any
tranfparent Subftance, if it be formed into a fpherical

---

[a] *Philofoph. Tranfact.* No. 267. *pag.* 722.

or cylindrical Figure. For this purpose therefore I
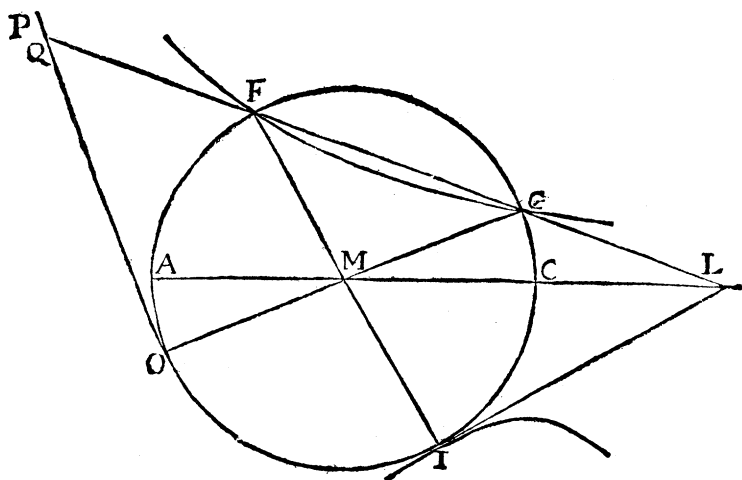
have found, that in the latter Determination of the
firft Propofition, if the Sine of the Angle under C MG
be denoted by *a*, the Tangent of the Complement of
this Angle to a right one be denoted by *b*, and the Se-
cant of this Complement by *c*; the Root of this E-
quation $z^3 - 3aaz = 2aa \times 2c - a$ will exceed
the Sine of the Angle under F M A, that is the Sine
of the Angle of Incidence, by the Sine of the Angle
under C M G ; and the Sine of the Angle under FMO,
which is double the refracted Angle, will be the Root
of this Equation $x^3 + 3aax = 4aab$; this Angle
being acute, when the Tangent of the Angle under
C MG is lefs than half the *Radius*, or when the
Angle itfelf is lefs than 26 *degr.* 33′. 54″. 11‴, and
when this Tangent is more than half the *Radius*, the
Angle under O M F is obtufe.

The

The Roots of thefe cubic Equations are found by feeking the firft of two mean Proportionals, between each of the verfed Sines appertaining to the Arches C G, A G, and the Sine of thofe Arches, counting from the verfed Sines; for the Sum of thefe two mean Proportionals is the Root of the former Equation, and the difference between them the Root of the latter ; as may be collected from *Cardan's* Rules.

And hence likewife if the firft and laft of the five mean Proportionals, between the Sine and Cofine of half the Angle under C M G be found, twice the Sum of the Squares of thefe mean Proportionals applied to the *Radius* exceeds the Sine of the Angle of Incidence by the Sine of the Angle under C M G ; and twice the difference of the Squares of the fame mean Proportionals applied to the *Radius* is equal to the Sine of double the refracted Angle. Moreover this double of the refracted Angle exceeds the Angle of Incidence by the Angle under C M G.

In the latter Determination of the fecond Propofition draw K Y, and A Y being parallel to M N, the Angle under C K Y will be equal to twice the Angle under

under C M N, that is equal to the Complement of half the Diſtance of the exterior Rainbow from the Point oppoſite to the Sun. Then putting *a* for the *Radius* A K, and *b* for the Sine of the Angle under C K Y, the Sine of the Angle under A K V will be the Root of this Equation $y^4 + 4by^3 - 8aaby + 4aabb = 0$. But the Angle of Incidence and Refraction may alſo be found as follows.

Let two mean Proportionals between the *Radius* and the Sine of the Angle under C K Y be found, then take the Angle, whoſe Coſine is the firſt of theſe mean Proportionals, counting from the *Radius*; and alſo the Angle, whoſe Sine together with the ſecond mean Proportional ſhall be to the *Radius* as the Coſine of the Angle under C K Y to the Sine of the Angle before found. The Sum of theſe three Angles is double the Complement to a right one of the Angle under A K L, the Angle under K M L, or the refracted Angle, being equal to half the Sum of this Angle under A K L and the Angle under C K Y; as in the laſt Place the Angle under K L V, that is the Angle of Incidence, equal to the Sum of the Angles under K M L and under M K L.

I need not obſerve, that the geometrical Methods of deducing theſe Angles of Incidence and Refraction from the Angle meaſuring the Diſtance of each Rainbow from the Point oppoſite to the Sun, afford very expeditious mechanical Conſtructions.